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Modifying the normalized Fry method for aggregates of non-elliptical grains

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Abstract—A modification of the normalized Fry method is developed to avoid the problems associated with approximating non-elliptical grains by best-fit ellipses. This modified method is a more general approach for constructing a normalized Fry plot and determining the orientation and axial ratio of the fabric ellipse. Grains can be approximated by any shape as long as the center and area of each grain can be determined. Irregular polygons work particularly well since they can be used to approximate both elliptical and non-elliptical grains.

Once the center and area of each grain in an aggregate is calculated, existing methods can be used to construct a normalized Fry plot. Determining the fabric ellipse, however, requires a new approach. A circular grid is used to find points on the ring of maximum point density along a series of radial vectors. The least-squares best-fit ellipse calculated through these points gives an initial estimate of the fabric ellipse. An iterative approach is taken, using the fabric estimate to calculate a plane strain irrotational inverse deformation matrix which, in turn, is used to unstrain grain centers. The newly determined centroids are used to calculate a new fabric ellipse. This process is repeated until the axial ratio of the remaining fabric ellipse is 1.00. The fabric ellipses for each stage are combined to calculate the total fabric ellipse.

Tests of this method using computer generated data sets of elliptical and non-elliptical grains show a strong correlation of measured fabric with known artificially imposed fabric.

INTRODUCTION

FINITE strain analysis in naturally deformed rocks is carried out by measuring markers in the deformed state that had known, or assumed, shapes in the non-deformed state (Ramsay 1967, Ramsay & Huber 1983). The use of strain analysis for quantifying deformation is usually limited by the availability of adequate strain markers. One group of abundant markers in sedimentary rocks are aggregates of grains such as sandstones and oolitic limestones. The major problem with using such aggregates for strain analysis is finding an adequate method, particularly for aggregates of non-elliptical shape grains.

Fry (1979) developed a simple and powerful technique for determining strain from aggregates of grains based on the distribution of object centers. Object center-to-center distances plotted on a radial diagram produce three general regions: (1) an inner void where points are excluded because grain boundaries cannot overlap; (2) a maximum point density ring where nearest neighbor grains plot; and (3) an outer region of lower density point concentrations where non-nearest neighbor center-to-center distances lie. The maximum point density ring represents the fabric ellipse (Fig. 1a).

Erslev (1988) recognized that the variability of two-dimensional grain size causes scatter on a Fry plot, making it difficult to locate a distinct maximum point density ring. He overcame this problem by normalizing the distance between two-dimensional grain centers to the sum of their average radii. For ideal non-deformed samples the normalized center-to-center distance of nearest neighbor grains is 1.0, no matter what the size of the grains. Since the initial value is constant, a sharp maximum point-density ring marks the fabric ellipse (Fig. 1b).

The most general approach for constructing a normalized Fry plot using Erslev's (1988) method currently requires approximating each grain in an aggregate by an ellipse. The ellipse is found by calculating the least-squares best-fit ellipse (Erslev & Ge 1990) to equally spaced points along grain boundaries. The ellipse serves three functions: (1) the center of the ellipse represents the center of the grain and is used in calculating the center-to-center distances; (2) the area of each ellipse is used to calculate the average radius ($\sqrt{\text{area}/\pi}$) of each grain which is used to normalize the center-to-center distance; and (3) the boundary of each ellipse is used to locate nearest neighbor (touching or nearly touching) grains. The center-to-center distances of nearest neighbor grains define the maximum point-density ring of normalized Fry plots. By excluding non-nearest neighbor grains. Erslev & Ge (1990) were able to separate the maximum point density ring from other points on the Fry plot, creating what they called an enhanced normalized Fry plot (Fig. 1c). The least-squares best-fit ellipse to the points on the enhanced plot represents the fabric ellipse.

While the approach of Erslev & Ge (1990) works well for approximating elliptical grains, serious problems can occur when trying to approximate non-elliptical grains by ellipses (Fig. 2). Another method should be used on aggregates of non-elliptical grains. However this would mean using the original Fry method rather than the normalized Fry method and accepting the scatter due to variation in grain size. The challenge, therefore is to develop a method that avoids both the problems associated with the elliptical approximation of grains and the

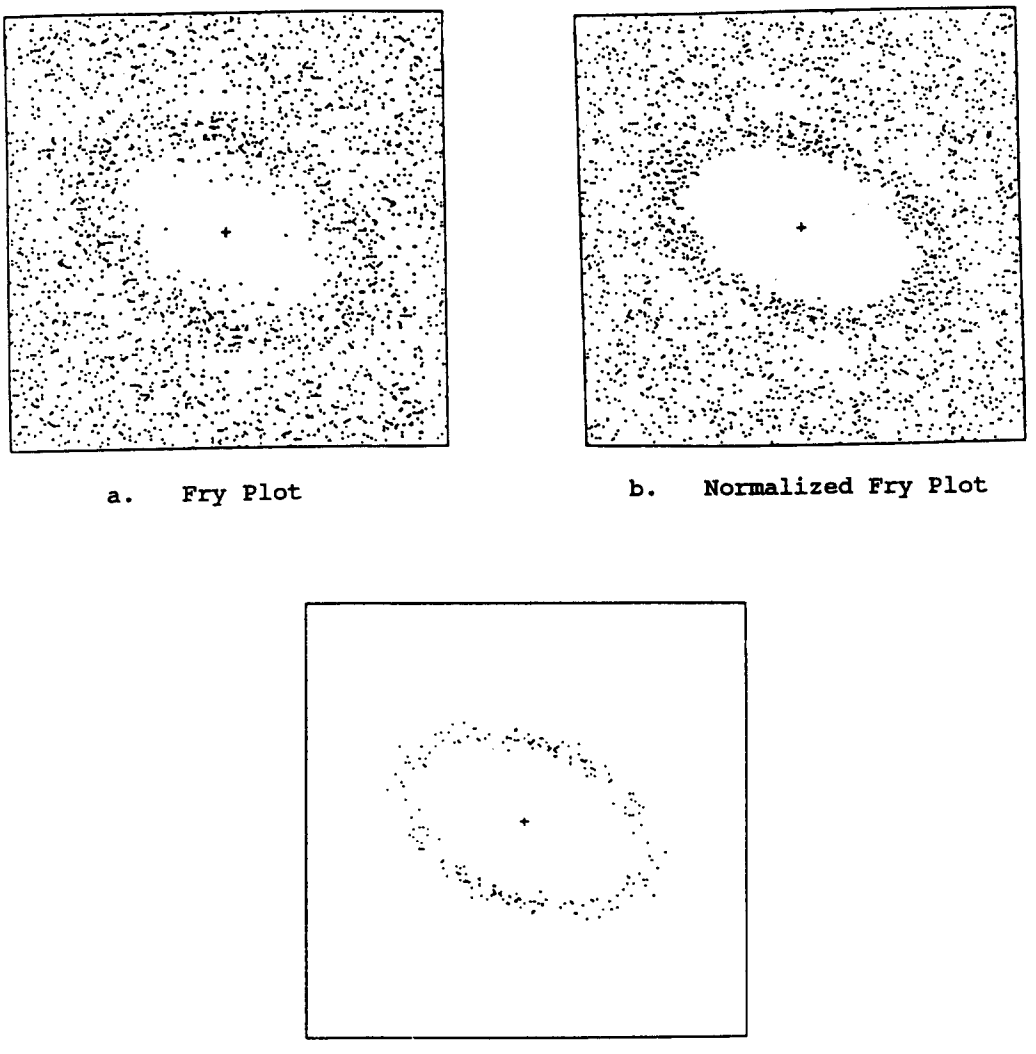

Fig. 1. Fry and normalized Fry plots of fig. 7.7 of Ramsay & Huber (1983) constructed using the method of Erslev (1988) and Erslev & Ge (1990). Each grain was approximated by a least-squares best-fit ellipse. (a) Conventional Fry plot, (b) normalized Fry plot and (c) enhanced normalized Fry plot.

problems associated with the variation in two-dimensional grain size, and still allow the use of aggregates of deformed grains for finite strain analysis. This can be achieved by approximating individual grains by polygons rather than ellipses. Approximating grains by polygons requires modifying the method by which the maximum point density ring is used to determine the fabric ellipse. This approach turns out to be a more generalized approach for constructing a normalized Fry plot. It works equally well for objects approximated by ellipses as it does for objects approximated by irregular polygons.

METHOD

Approximating grain boundaries

Starting with a photomicrograph or a negative print of a thin section, grain boundaries are traced onto an overlay. The overlay is placed on a digitizing tablet where points along the grain boundaries are entered into a computer file. Points can be entered either by having the computer record equally spaced points during tracing of the grain boundary, or by manually selecting individual points in series along the boundary.

Connecting these points in series by straight-line segments form irregular polygons that approximate each grain. The points are the vertices of the polygons. This approximation is valid for grains with straight boundaries or grains with curved boundaries if the spacing between vertices is kept small. Such an approximation was used by Panozzo (1983, 1984) for determining strain from grain shape; here the approximation is used for determining strain from center-to-center distances.

Once each grain is approximated by a polygon the center and area of that polygon is calculated. The center and area of the polygon are sufficient to construct a normalized Fry plot. Computer programs for calculating the center and area of polygons from their digitized vertices have already been published (for example see Zarkos & Rodgers 1987, McNaught 1991).

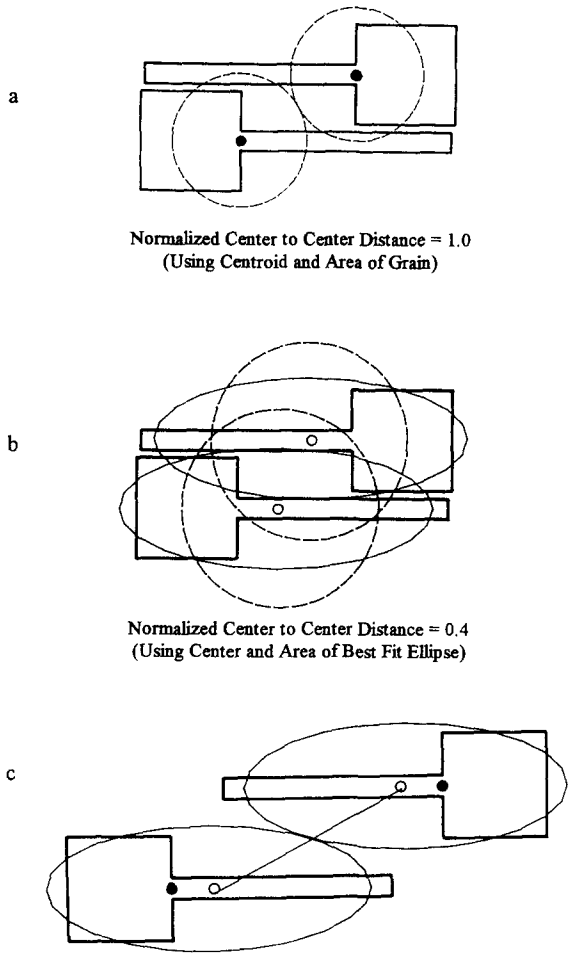

Fig. 2. (a) Two idealized non-elliptical grains. Closed circles mark centroid of each grain. Large dashed circles indicate the length of the average radius of each grain determined from the area of each grain. Normalized center-to-center distance based on center and average radius of polygon is 1.0. (b) Same grains as (a), but with the least-squares best-fit ellipse and center of best-fit ellipse (open circle) shown. Large dashed circles indicate the length of the average radius of each grain determined from the area of the best-fit ellipse. Note that the center of the best-fit ellipse does not correspond to the centroid of the grain as seen in (a). Normalized center-to-center distance based on center and average radius of best-fit ellipse is 0.4. This is significantly different than in (a). (c) Difference in determining nearest neighbor grains for elliptical and polygonal approximations of grain boundary. The two grains are nearest neighbors based on touching best-fit ellipses, but are not based on actual irregular polygon shape which do not touch.

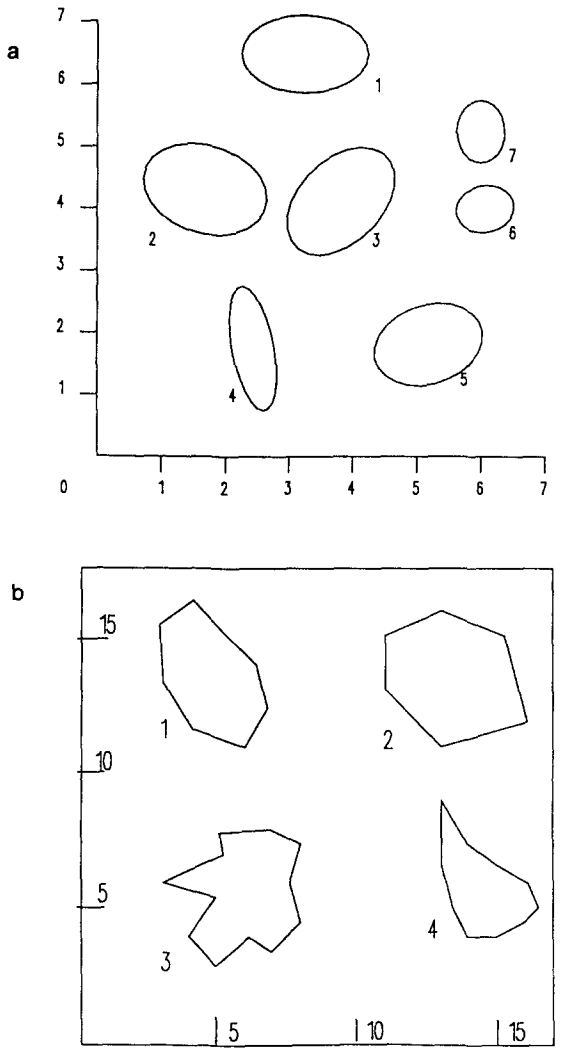

Fig. 3. (a) Ellipses used for testing grain approximations. See Table 1. (b) Polygons used for testing grain approximations. See Table 2.

The validity of approximating grain boundaries by polygons can be tested by comparing least-squares best-fit ellipse and irregular polygon approximations calculated from the same points (Fig. 3). Both approaches produce similar results for elliptical objects (Table 1). For polygonal objects the results depend on the object shape (Table 2). For some shapes (polygonal objects 3 and 4 in Fig. 3b), the center and area of the best-fit ellipse deviates significantly from the exact centroid of each object (Fig. 4). The center and area of the irregular polygon used to approximate the object, however, closely matches the exact center and area of the object (<5% deviation, Fig. 4). Note that the irregular polygon used to approximate the object is not identical to the object because points digitized along the object's boundary do not necessarily correspond to the actual vertices of the shape.

This comparison of two methods for approximating grain shape shows that the elliptical approximation is subject to more variations due to irregular grain shapes than the polygonal approximation. Therefore, it is in general better to approximate grains by irregular polygons in order to determine center and area of each grain.

Constructing a normalized Fry plot

Given the center and area of each grain a normalized Fry plot can be constructed by the same method as used by Erslev (1988). The average radius of each polygon is defined the same way as the average radius of an ellipse ($\sqrt{\text{area}/\pi}$). The distance between each pair of grain centers is divided by the sum of each grain's average radius and plotted on a radial diagram in the direction of the line connecting the center of the two grains. The result of plotting normalized center-to-center distances of all possible grain pairs is a classic Fry type diagram with a distinct maximum point density ring representing the fabric ellipse.

Table 1. Centers and areas of the seven ellipses in Fig. 3(a) determined by least-squares best-fit ellipse and irregular polygon approximations

		Best-fit ellipse approximation			Irregular polygon approximation		
		Center		Area	Center		Area
	N	X	Y		X	Y	
Ellipse 1	28	3.23	6.49	1.95	3.23	6.51	1.95
Ellipse 2	27	1.63	4.32	2.17	1.63	4.33	2.14
Ellipse 3	27	3.75	4.14	2.08	3.75	4.15	2.06
Ellipse 4	23	2.34	1.76	1.04	2.33	1.78	1.02
Ellipse 5	24	5.09	1.84	1.71	5.09	1.85	1.69
Ellipse 6	13	6.00	4.02	0.54	6.00	4.02	0.51
Ellipse 7	14	5.96	5.25	0.58	5.96	5.26	0.56

Table 2. Centers and areas of the four polygons in Fig. 3(b) determined by exact centroid, least-squares best-fit ellipse and irregular polygon approximation. Offset from the exact centroid is expressed as a percentage of the average radius. Difference in area is expressed as a percentage of the exact area. N is the number of equally spaced points digitized along the boundary of each polygon

	N	Center		Area	Offset	
		X	Y		Center	Area
(a) Exact centroid						
Polygon 1		4.75	13.51	13.2		
Polygon 2		13.38	13.51	17.2		
Polygon 3		6.02	5.65	14.4		
Polygon 4		14.44	5.80	8.6		
(b) Best-fit ellipse						
Polygon 1	26	4.71	13.42	13.3	4.9	0.7
Polygon 2	28	13.40	13.37	17.6	5.6	2.3
Polygon 3	31	5.70	5.64	16.8	14.8	16.7
Polygon 4	23	14.43	6.11	10.6	18.9	23.2
(c) Irregular polygon approximation						
Polygon 1	26	4.76	13.46	12.8	3.9	−3.0
Polygon 2	28	13.39	13.48	17.0	4.5	−1.2
Polygon 3	31	6.09	5.72	14.2	4.8	−1.4
Polygon 4	23	14.44	5.81	8.3	0.6	−3.5

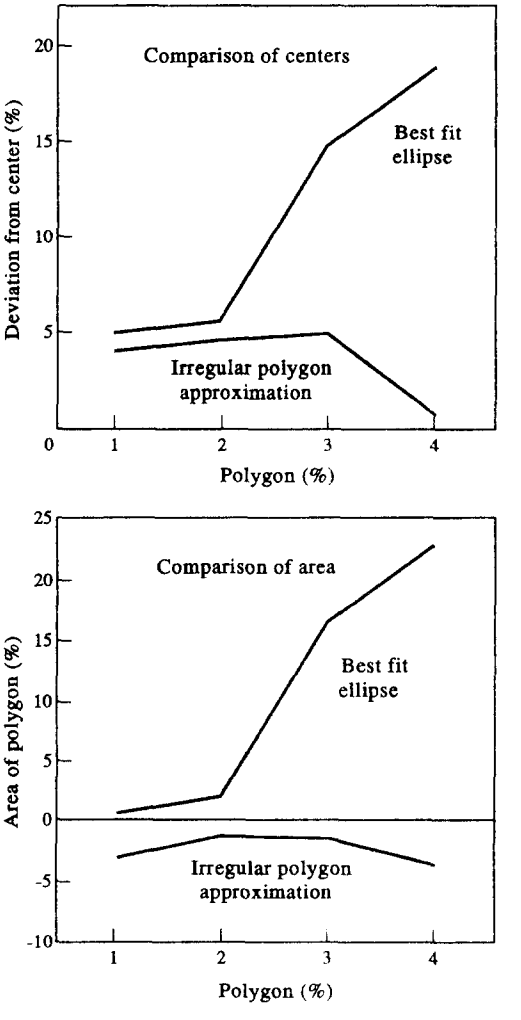

Fig. 4. (a) Deviation of the center determined by the least-squares best-fit ellipse approach and the centroid of the irregular polygon approximation, from the calculated centroid of the polygons in Fig. 3(b). Deviation is expressed as a percentage of the average radius of each polygon. (b) Deviation of area determined by the least-squares best-fit ellipse approach and irregular polygon approximation, from the calculated areas of polygons in Fig. 3(b). Deviations are expressed as a percentage of the actual area of each polygon.

Determining the fabric ellipse

In order to determine the axial ratio and orientation of the fabric ellipse it is necessary to fit an ellipse to the maximum point density ring of the Fry plot. While it is possible to pick points on the fabric diagram visually (i.e. De Paor 1989), an automated method is more desirable. Automated fabric ellipse fitting requires separating points of the maximum point density ring from the surrounding background points.

A disadvantage in using irregular polygons to approximate grain boundaries is the difficulty in determining nearest neighbor grains. While an ellipse can be represented by a single equation, each side of an irregular polygon needs to be expressed by a linear equation. Since these polygons can have 20–50 sides, locating nearest neighbor grains require manipulating 40–100 equations for every possible pair of grains. This is not practical. Instead the non-enhanced normalized Fry plot must be analyzed by a different method.

For any given radial direction on a Fry plot, the location of maximum point density represents a point on the fabric ellipse. These points can be located by dividing the plot into a series of sectors with a constant angular spacing. Each sector is divided into a series of regions by equally spaced arcs. Note that the length of each region is expressed as a normalized (dimensionless) distance. The region with the highest point density determines the point on the fabric ellipse for that sector. Regions overlap both longitudinally and radially (Fig. 5). For example one sector may range from 30° to 60°, the next 40°–70° and next 50°–80° (offset of 10°). Within

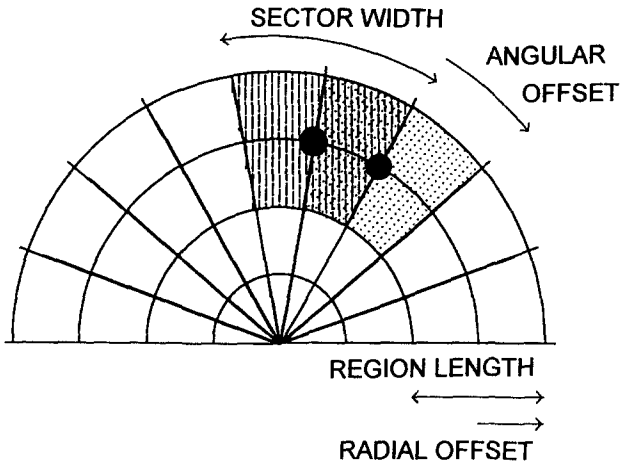

Fig. 5. Illustration of part of the overlapping grid. Large solid circles mark the center of two adjacent grid regions, one region is shaded by dotted lines and the other is shaded by dashed lines. Note the area between the two region centers belongs to both regions. Points lying in this area are counted in the totals for both regions.

each sector one region may range from 0.5 to 0.8, the next 0.55–0.85 and the next 0.6–0.9 (offset of 0.05). This is what is termed the overlapping grid. A single point therefore lies in a series of grid localities. This prevents a grid boundary from arbitrarily cutting a maximum density area into two parts. Once the maximum density concentration is found in any specified direction, the radial values within the region are averaged to determine the value of a point on the fabric ellipse. The least-squares best-fit ellipse (Erslev & Ge 1990) to the points found from each sector is the fabric ellipse.

A serious drawback of using a circular grid to calculate points on an elliptical point density ring is the tendency for fabric to be underestimated, especially at higher axial ratios. This problem can be avoided by taking an iterative approach which removes the calculated fabric by retrodeforming the calculated centroid of each grain.

An inverse deformation matrix for irrotational strain can be calculated by modifying equation (C-17) of Ramsay & Huber (1983):

$$\begin{Bmatrix} \sqrt{\frac{1}{R}}\cos^2\phi' + \sqrt{R}\sin^2\phi' & \left[\frac{\frac{1}{R}-1}{1/\sqrt{R}}\right]\sin\phi'\cos\phi' \\ \left[\frac{\frac{1}{R}-1}{1/\sqrt{R}}\right]\sin\phi'\cos\phi' & \sqrt{R}\cos^2\phi' + \sqrt{\frac{1}{R}}\sin^2\phi' \end{Bmatrix}. \quad (1)$$

Given the initial approximation of the fabric ellipse, R_a, ϕ_a the first inverse deformation matrix can be calculated by substituting R_a and ϕ_a^* for R and ϕ' in equation (1). This matrix is used to multiply the co-ordinates of grain centroids to produce unstrained centroid co-ordinates.

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = [\text{Inverse} \quad \text{Deformation}] \begin{bmatrix} X_{i-1} \\ Y_{i-1} \end{bmatrix}. \quad (2)$$

If the first estimate accurately determines the fabric ellipse, then the new maximum point density ring is a circle. If the initial estimate is too low, the new maximum point density ring is elliptical. Using the newly determined values of R and ϕ in equation (1), a second inverse deformation matrix is calculated in order to adjust the polygon centers using equation (2). This process is repeated until a circular maximum point density ring is found. As a practical limit for this study axial ratios are rounded off to two decimal places so ellipses with axial ratios less than 1.005 are considered circular.

While the centers are being adjusted, each inverse deformation matrix is pre-multiplied by the previous inverse deformation matrix:

$$\begin{bmatrix} \text{Inverse} \\ \text{Deformation} \\ \text{Total} \end{bmatrix} = \begin{bmatrix} \text{Inverse} \\ \text{Deformation}_i \end{bmatrix} \begin{bmatrix} \text{Inverse} \\ \text{Deformation}_{i-1} \end{bmatrix} \cdots \begin{bmatrix} \text{Inverse} \\ \text{Deformation}_1 \end{bmatrix} \begin{bmatrix} \text{Identity} \end{bmatrix}. \quad (3)$$

When a circular maximum point density ring is found the values of axial ratio and orientation of the reciprocal strain ellipse (R_r, ϕ_r) can be found using equations (B-14) and (B-20) of Ramsay & Huber (1983):

$$\tan 2\phi_r' = \frac{2(ac+bd)}{a^2+b^2-c^2-d^2} \quad (4a)$$

$$R_r = \left(\frac{a^2+b^2+c^2+d^2+[(a^2+b^2+c^2+d^2)^2-4(ad-bc)^2]^{1/2}}{a^2+b^2+c^2+d^2-[(a^2+b^2+c^2+d^2)^2-4(ad-bc)^2]^{1/2}}\right)^{1/2}, \quad (4b)$$

where a, b, c and d are the four components of the total inverse deformation matrix. Values of the axial ratio and orientation of the fabric ellipse are given by $R = R_r$ and $\phi' = \phi_r' + \pi/2$.

Setting grid parameters

One problem with computer-based Fry techniques is that the determined fabric ellipse depends on some arbitrarily set parameters. For example using the method of Erslev & Ge (1990), the selection factor specified affects the fabric ellipse that is determined. The overlapping grid approach presented here also suffers from this problem since it has five parameters that need to be specified: grid size, sector width, region length, angular offset and radial offset. The effects of these parameters have not yet been fully investigated but some guidelines can be offered for choosing proper values.

To set the overall grid size a specified radius is required. While it is possible to chose this radius so that all normalized center-to-center distances will lie within the grid this would require calculating point densities far outside the maximum density ring. To avoid such tedious computation, the grid radius can be limited so it exceeds the radius of the maximum point density ring but is less than the largest center-to-center distances. Since the area of the fabric ellipse is π in normalized space, the maximum fabric ellipse axial ratio that can be

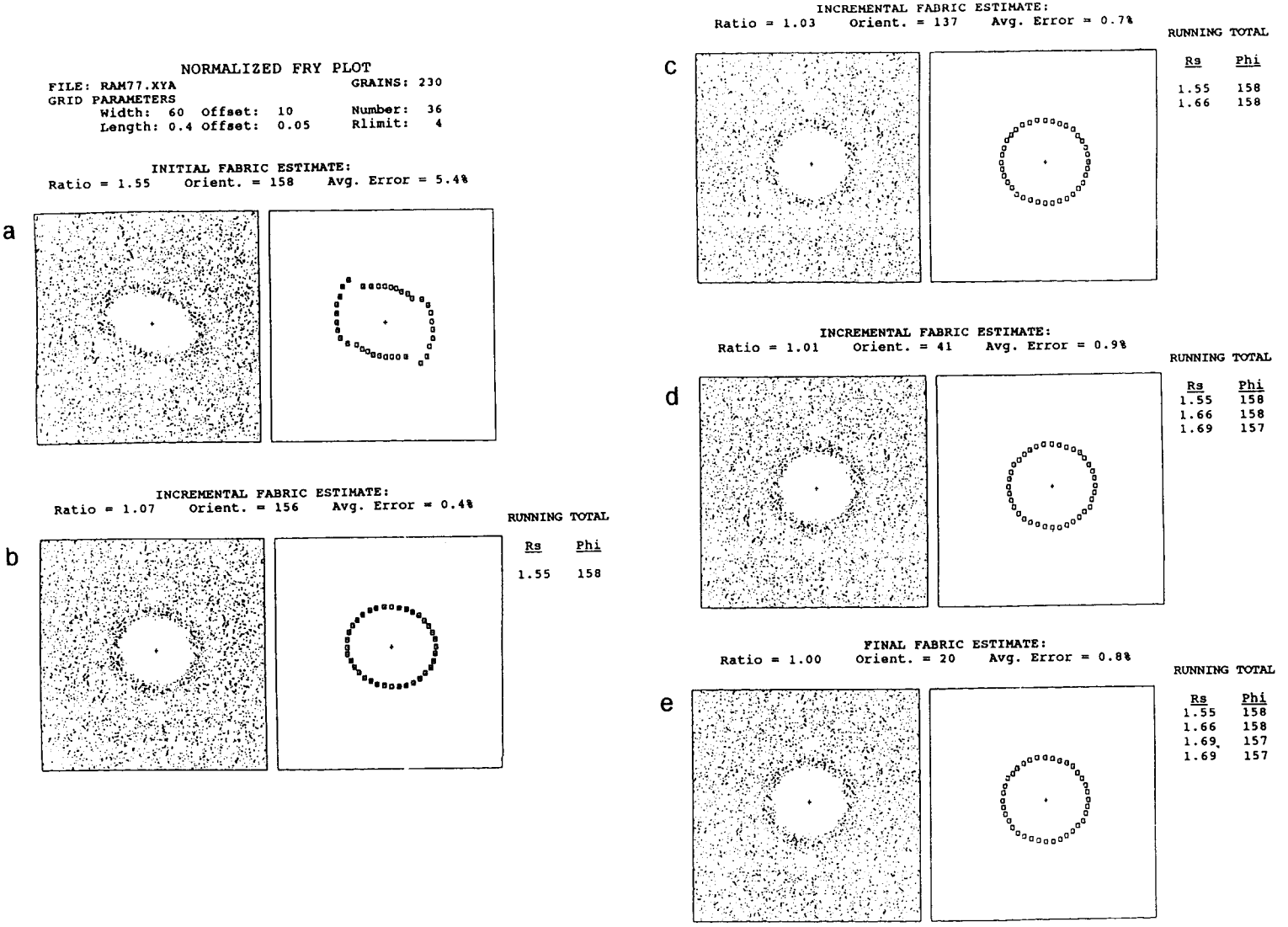

Fig. 6. Normalized Fry plot constructed from fig. 7.7 of Ramsay & Huber (1983) using overlapping grid method. For each diagram normalized Fry plot is shown on the left and points determined to be on the fabric ellipse are shown on the right. Axial ratio and orientation of fabric ellipse for each increment is shown above plot. Running total of strain is given in the right-hand columns. (a) Initial estimate of the fabric ellipse. (b)–(d) Successive estimate of the fabric ellipse after each deformation reversal. (e) Final fabric ellipse ($R = 1.69$, $\phi = 157°$) is determined when estimate of the measured incremental fabric is 1.00.

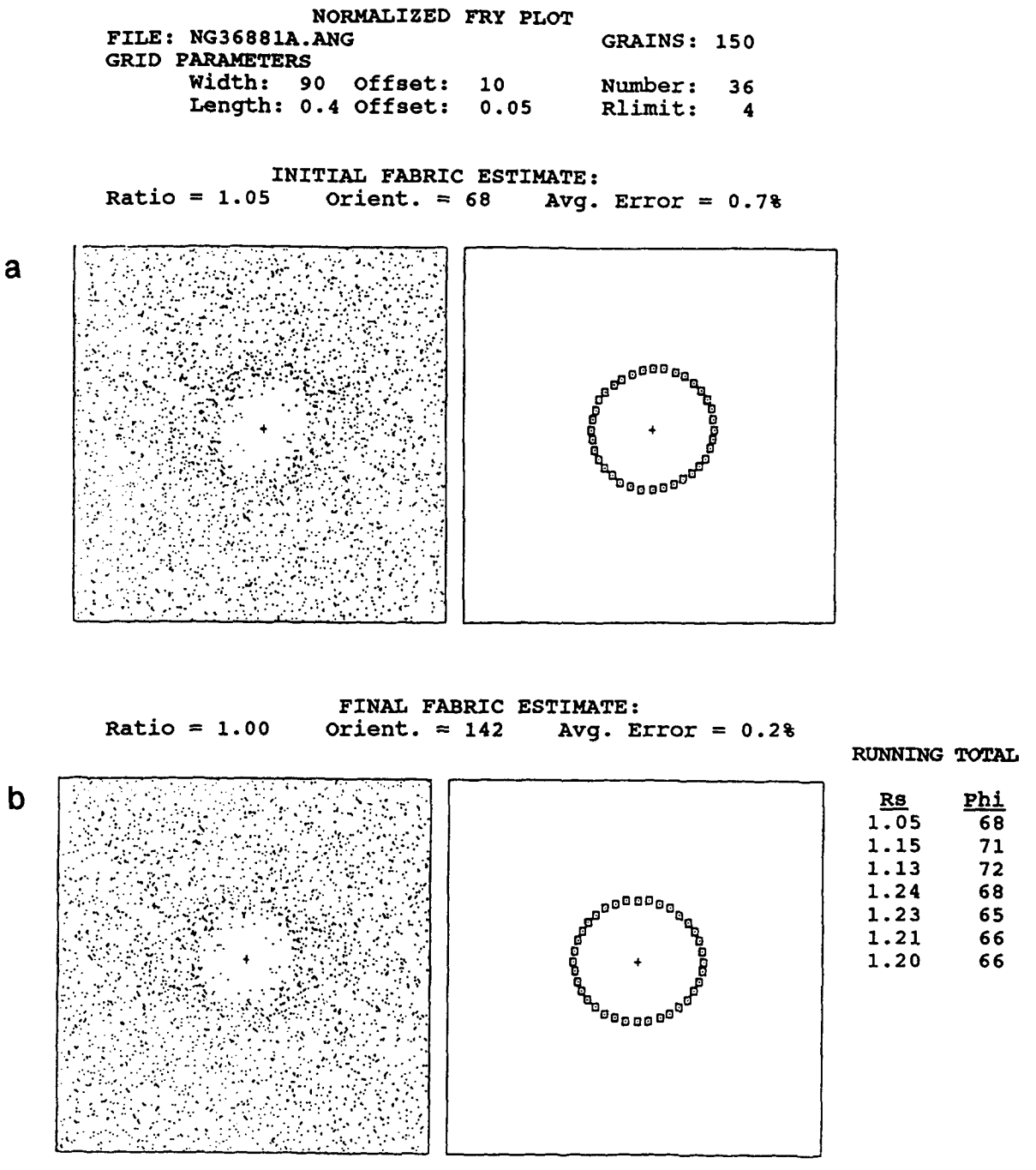

Fig. 7. Normalized Fry plot constructed by approximating sandstone grains by irregular polygons. (a) Initial estimation of fabric. (b) Final estimate shows no remaining fabric and total fabric of $R = 1.20$, $\phi = 66°$.

determined is the square of the radial limit. For example the radial limit of 4 used in this study should satisfy fabric ellipse ratios of up to 16:1. Since the fabric ellipse is the unknown and the maximum point density ring always has some associated scatter, it is best to leave considerable leeway in choosing grid size.

The angular offset of each sector specifies the number of points that will be used to determine the fabric ellipse. While smaller angular offsets allow more points to be used for calculating the fabric ellipse, the calculation time is increased accordingly. For all of the analyses preformed for this paper an angular offset of 10° was used.

The radial offset of each region needs to be kept small relative to the length of each grid region so there is sufficient overlap to avoid cutting the maximum density ring. Small offsets increase the number of regions in each sector resulting in slower speed of calculation. For this study radial offsets have been set at 0.05.

The radial length of each grid region needs to exceed the width of the maximum point of density ring. Values of 0.4 were used for all the Fry plots in this study.

The angular width of each sector is the most important variable. While narrow sectors give a better initial estimation of the fabric they often mistake spurious local point concentrations for the maximum point density ring. Wider sectors effectively use more points to locate the fabric ellipse, so local point concentrations have

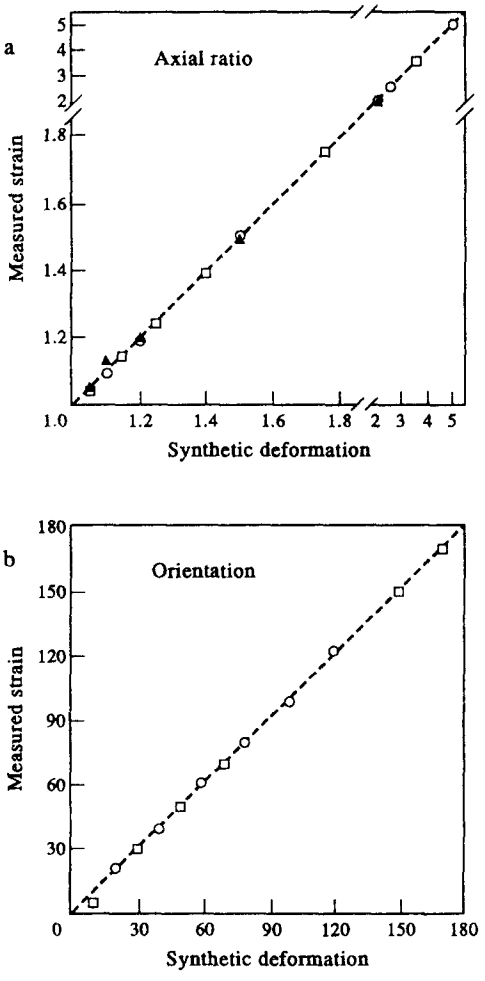

Fig. 8. Comparison of measured fabric with imposed synthetic deformation. Open squares are data set sesrt1 (382 random ellipse with axial ratios between 1 and 1.5) derived from fig. 11(a) of Erslev & Ge (1990). Open circles are data set rdt1 (408 random ellipse with axial ratios between 1 and 3) derived from fig. 11(b) of Erslev & Ge (1990). Solid triangles are for population of irregular polygons shown in Fig. 10 whose centroid and area was determined by an image analyzer. (Note that orientation was kept constant during synthetic deformation so only axial ratios are plotted.) Diagonal lines represent 1:1 ratio. All populations show strong correlation between measured strain and synthetic deformation.

little effect. Wider sectors however also cause lower initial estimates of the fabric ellipse, but iteratively undeforming the Fry plot and recalculating the ellipse makes up for this initial underestimation. For this study the chosen sector width depends on the sharpness of the maximum point density ring and the number of grains used for the plot. For well-behaved synthetic data with greater than 350 grains, sector widths of 20° are appropriate. For data sets with a less distinct maximum point density ring and fewer points, larger sector widths are needed. In general, if there is scatter among the points chosen on the maximum point density ring or if the Fry plot cannot be retrodeformed, the width of the sector should be increased.

RESULTS

The results of fabric analysis on two natural aggregates illustrate the method outlined above. As a first example the data set used to construct Fig. 1 (Ramsay & Huber 1983, fig. 7.7) was used to calculate centers and area of grains and used in turn to construct a normalized Fry plot (Fig. 6a). Using a radial grid to locate points on the fabric ellipse, the initial estimate of the axial ratio and orientation is 1.55 and 158°. These values are used to calculate an inverse strain matrix (equation 1) in order to retrodeform the co-ordinates of each grain's center (equation 2). The new center co-ordinates are used, in turn, to construct a new normalized Fry plot (Fig. 6b). Since the new Fry plot has a fabric ($R = 1.07$, $\phi' = 156°$) the retrodeformation process is repeated until the incremental fabric ellipse is a circle (Fig. 6e). For each step the fabric estimates are combined to keep a running total (equation 3). At the stage when the incremental fabric ellipse is a circle, the running total gives the axial ratio and orientation of the fabric ellipse for the sample. In this case $R = 1.69$ and $\phi' = 157°$. Notice that orientation of the long axis of the fabric ellipse is nearly constant for each stage.

Figure 7 shows a normalized Fry plot calculated for a sandstone by approximating grains as irregular polygons. The centroid and area of each polygon was used to construct a normalized Fry plot (Fig. 7a). The initial estimate of the axial ratio of the fabric ellipse is 1.05 and orientation is 68° (Fig. 7a). The sample is unstrained until no remaining fabric is detected. The axial ratio of the fabric ellipse is 1.20 with long axis oriented 66° (Fig. 7b).

To test the method, the synthetic aggregates illustrated in fig. 11 of Erslev & Ge (1990) were used to construct data sets with known fabric. These populations of ellipses were rotated and copied so that the computer-generated data set had no initial fabric. The calculated center of each ellipse was deformed by plane strain with known axial ratios and orientations. Figure 8 shows that the measured strain closely matches the synthetic deformation.

A population of polygons was generated in a similar fashion (Fig. 9). Polygons were drawn free-hand using a CAD program in a 20° wide sector so that the left and right boundaries of the sector match. This 20° sector was then rotated and copied around the center of the diagram to ensure that no initial fabric was present. The diagram was distorted to known axial ratios on the computer and plotted. Rather than digitizing boundaries, centroids and areas were found using an image analyzer. Normalized Fry plots were then constructed by the method described above (Fig. 10). As with the test ellipses above, the final measured strain was an accurate determination of the synthetic deformation (Fig. 8).

The significance of the axial ratio and orientation of the measured fabric ellipse was estimated by using a bootstrap technique. Bootstrapping involves creating artificial data subsets by randomly choosing objects from the original data set (Efron & Tibshirani 1986, Ratliff in press). Each subset has the same number of objects as the original but is different because objects can be chosen once, more than once, or not at all. The standard

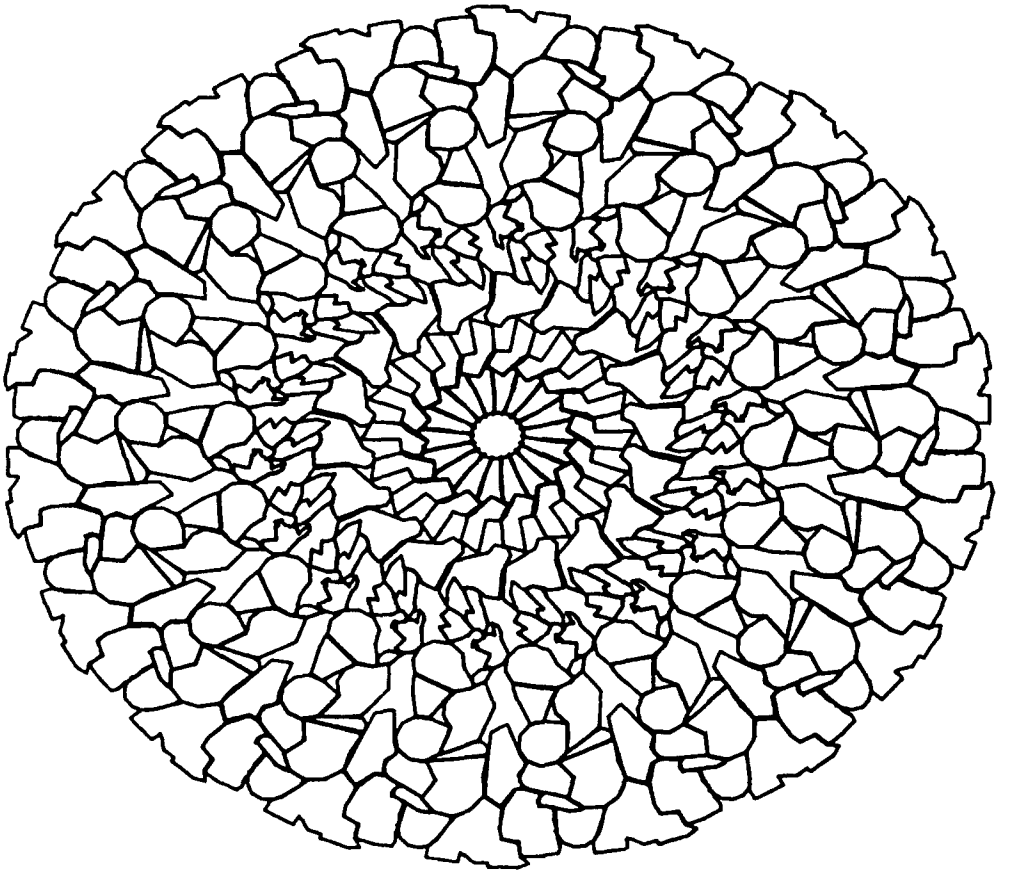

Fig. 9. Example of idealized irregular polygon population used to test image analyzer. Synthetic strain = 1.20.

deviation of the results from analyzing all the data subsets approximates the standard deviation of the original data set (Ratliff personal communication 1992).

For this study the axial ratio and orientation of the fabric ellipse were determined for 100 subsets created for each of the six data sets by randomly choosing grains. Two of the subsets are from the synthetically deformed random ellipses taken from Erslev & Ge (1990) that were discussed above. Three of the data sets are of the synthetic polygons illustrated in Fig. 10 with different axial ratios. The final data set is from fig. 7.7 from Ramsay & Huber (1983).

The axial ratio and orientation of the fabric ellipse were found for each data subset. Note that some subsets would not retrodeform in fewer than 20 increments and were discarded. The standard deviation of the 100 R and ϕ' values were calculated for each original data set (Table 3).

Mean values agree closely with the imposed synthetic deformation. The standard deviation for the axial ratio is higher for the polygonal objects than for the elliptical objects. It is also higher at higher axial ratio for the same set of objects. The standard deviation for the orientation of the fabric ellipse is also higher for the polygonal data sets and higher at lower axial ratio. Though the number of data sets analyzed is small, the preliminary results indicate that, for axial ratios, the second decimal place is not significant.

DISCUSSION

A comparison of polygonal based and elliptical based normalized Fry analysis is difficult because of limitations of the artificial standards used. While problems associated with approximating non-elliptical grains by least-squares best-fit ellipses can be illustrated for individual grains (Fig. 2), it is difficult to show convincing discrepancies in Fry plots. The accuracy of a method is tested by comparing the fabric imposed on a population of computer-generated objects with the fabric measured from the same population. These computer-generated objects have an initial rotational symmetry to ensure no initial fabric. The rotational symmetry, however, repeats any anomalous center-to-center distances that might develop as a result of grain shape. Since these anomalous points are systematically located around the center of a Fry diagram, the effect of any one point on the fabric ellipse is balanced by the other anomalous points. Actual samples will not have rotational symmetry so anomalous points will tend to distort the fabric ellipse. But, since the fabric ellipse is unknown, it is impossible to tell how significant the distortion will be.

Despite this difficulty there are sound theoretical reasons to be concerned with approximating non-elliptical grains by ellipses. These problems can be avoided by using the method presented here. This method is quite flexible in terms of the type of data that

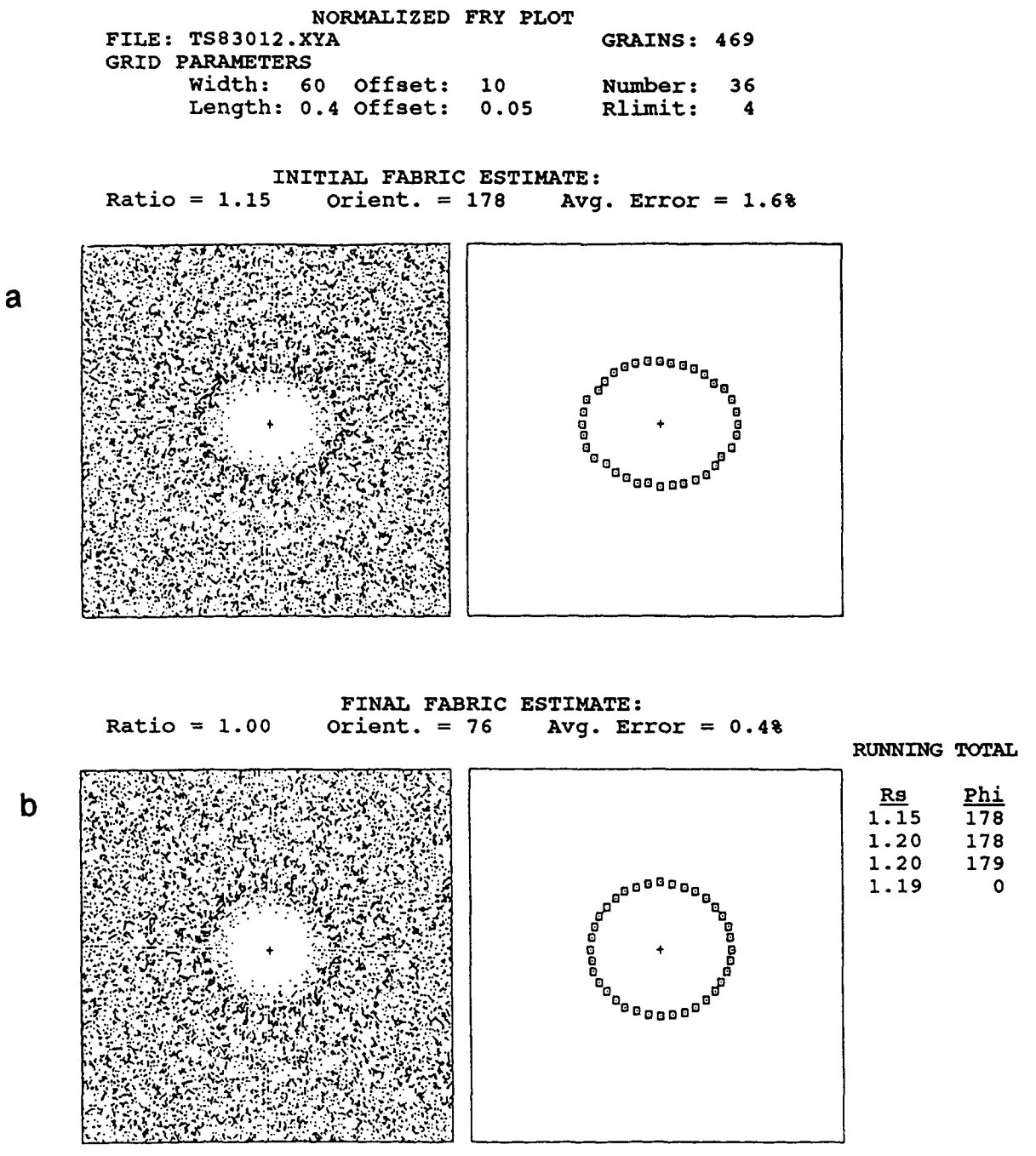

Fig. 10. Normalized Fry plot constructed using centroids and areas determined by an image analyzer from Fig. 9.

Table 3. Results of bootstrap analysis. R_{syn} and ϕ'_{syn} are axial ratio and orientation of the computer generated fabric. R and ϕ' are the axial ratio and orientation of the measured fabric ellipse

			Bootstrap results				
File	Type*	R_{syn}	ϕ'_{syn}	R	SD	ϕ'	SD
Sesrt14	se	1.4	50	1.392	0.043	50	2
Rdt15	se	1.5	60	1.493	0.050	60	2
TS83112	sp	1.2	180	1.195	0.090	177	22
TS83115	sp	1.5	180	1.519	0.137	178	13
TS83115	sp	2.0	180	1.910	0.163	178	7
Ram77	no			1.714	0.084	155	8

*se = synthetic ellipses; sp = synthetic polygons; no = natural oolite.

can be used since approximations only arise when determining the centers and areas of grains in an aggregate. As a result, grains can be approximated by ellipses, polygons or other shapes, as long as the center and area of the shape can be determined. Data files recorded to calculate least-squares best-fit ellipses can be used directly for calculating irregular polygons. Finally an image analyzer that specifies the centers and areas of grains in an aggregate adds considerable efficiency to the method.

This study indicates that results are independent of grid parameters. Consistent results are found for the same data set analyzed using different grid settings. While certain grid settings do not allow a data set to

successfully retrodeform in a reasonable number of increments (<20), modifying the grid settings solves this problem. Therefore, when a data set successfully retrodeforms there is no reason to believe that changing grid settings will significantly improve fabric estimate. The main effect of changing grid parameters is to optimize calculation time.

CONCLUSIONS

(1) Normalized Fry plots can be constructed by specifying the centroid and area of each grain in a population, either by approximating grains by ellipses or irregular polygons. Approximating grains by irregular polygons avoid some of the problems associated with approximating non-elliptical grains by least-squares best-fit ellipses. When a sufficient number of points are specified, an irregular polygon approximates an ellipse as effectively as a best-fit ellipse.

(2) Points on the fabric ellipse can be located by means of a radial grid. Incrementally removing the calculated fabric allows the circular grid to be used to calculate the fabric ellipse.

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